

Heat Transfer

28/3/2016

سید

conduction heat transfer with internal heat generation

AMR
M.r

4

sheet No. 2

Prob 4

Electrical cable

$$K = 190 \text{ W/m}^\circ\text{C}$$

$$D = 80 \text{ mm}$$

$$T_w = 180^\circ\text{C}$$

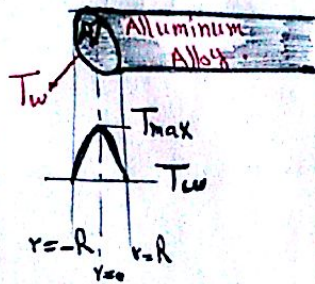
$$T_\infty = 15^\circ\text{C}$$

$$R = 15 \text{ mm}$$

$$I = 230 \text{ A}$$

$$\rho = 29 \mu\Omega\text{-cm}$$

resistivity.



Solution

$$T_{\max} = T_w + \frac{q_v \cdot R^2}{4K}$$

$\uparrow$ 180 $\rightarrow$ 190

Long solid cylinder

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = -\frac{q_v}{K}$$

$$\frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = -\frac{q_v}{K}$$

by integration twice we obtain

$$T = -\frac{q_v \cdot r^2}{4K} + c_1 \ln r + c_2$$

by using boundary conditions

(solid cylinder) we obtain

$$T = -\frac{q_v \cdot r^2}{4K} + T_w + \frac{q_v \cdot R^2}{4K} \quad \text{and} \quad c_1 = 0$$

$$T_{\max} = T_w + \frac{q_v \cdot R^2}{4K}$$

$$\dot{Q}_{\text{gen}} = \dot{Q}_{\text{conv}} = I^2 \cdot R_{\text{elec}}$$

$$R_{\text{elec}} = \frac{\rho \cdot L}{A_c}$$

$$\dot{Q}_{\text{gen}} = q_v \cdot \pi (0.015)^2 \cdot L = (230)^2 \cdot \frac{29 \cdot 10^{-8} \cdot L}{\pi (0.015)^2}$$

$q_v = 3070 \text{ W/m}^3$

$$R_{\text{elec}} = 29 \cdot 10^{-6} \cdot 10^{-2} = 29 \cdot 10^{-8} \Omega \cdot \text{m}$$

$$T_{\max} = 180.009^\circ\text{C}$$

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sheet No. 2

Prob 7

An electric wire $D = 10 \text{ mm}$? $k_w = 144 \text{ W/m} \cdot ^\circ\text{C}$

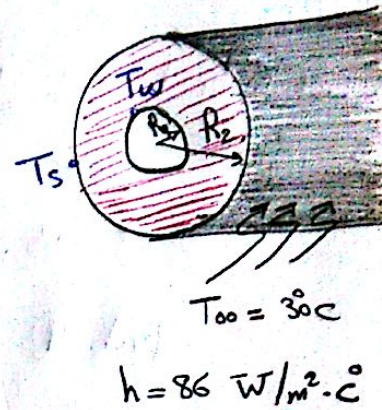
resistance $\rightarrow R_{elec} = 0.1 \Omega \cdot \text{m}$

thickness $\rightarrow \delta_{insulation} = 3 \text{ mm}$

$k_{insulation} = 0.15 \text{ W/m} \cdot ^\circ\text{C}$

$T_{max} = 150^\circ\text{C}$

$I_{max} = ? \text{ A}$



solution

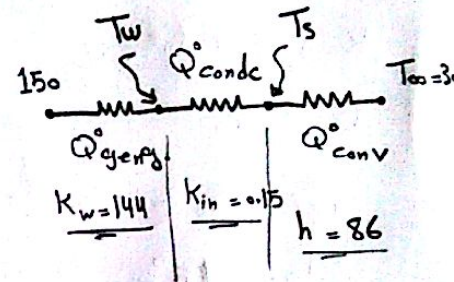
$$R_1 = 5 \text{ mm} = 0.005 \text{ m}$$

$$R_2 = 0.005 + 0.003 = 0.008 \text{ m}$$

$$Q_{gen} = I^2 R_{elec} = q_v \cdot V = \frac{q_v \cdot (0.005)^2 \cdot \pi \cdot L}{?}$$

معادلة توزيع درجات الحرارة المولدة

$$T_{max} = T_w + \frac{q_v \cdot R_1^2}{4 k_{wire} \text{ metal}} = 150 + \frac{q_v \cdot (0.005)^2}{4 \cdot 144}$$



1

For steady flow of heat.

$$Q_{gen} = q_v \cdot \pi R_1^2 L = \frac{(T_w - 30)}{\frac{\ln R_2/R_1}{2\pi k_{ins} L} + \frac{1}{h \cdot 2\pi R_2 L}}$$

2

$$T_w = \sqrt{\quad} \quad I = \sqrt{\quad}$$

حل 1 مع 2

$$q_v \cdot V = I^2 R_{elec}$$

$$I = \sqrt{\quad} \text{ A}$$

Prob 10

copper wire

$D = 2 \text{ mm}$

10

Environment ($h = 5000 \text{ W/m}^2\cdot^\circ\text{C}$)

$T_\infty = 100^\circ\text{C}$

$\rho_{\text{copper}} = 1.67 \times 10^{-6} \text{ } \Omega\cdot\text{cm}$

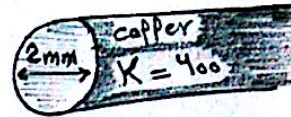
$$T_{\text{max}} = T_{\text{wire center}} = 150^\circ\text{C} \iff I = ? \text{ A}$$

From H.T. table for copper take $k_{\text{copper}} = 400 \text{ W/m}\cdot^\circ\text{C}$

Solution Commercial copper

$$R_{\text{elec}} = \frac{\rho \cdot L}{A_c}$$

$$R_{\text{elec}} = \frac{1.67 \times 10^{-6} \times 10^{-2} \times L}{\pi R^2}$$



$T_\infty = 100^\circ\text{C}$
 $h = 5000$

$$\dot{Q}_{\text{gen}} = \dot{q}_v \cdot \pi R^2 \cdot L = I^2 \cdot R_{\text{elec}}$$

$$T_{\text{max}} = T_w + \frac{\dot{q}_v \cdot R^2}{4k}$$

$$150 = T_w + \frac{\dot{q}_v \cdot (0.001)^2}{4 \cdot (400)} \rightarrow [1]$$

For steady flow of heat.

$$\dot{Q}_{\text{gen}} = \dot{q}_v \cdot \pi R^2 \cdot L = \dot{Q}_{\text{conv}} = h \cdot 2\pi R \cdot L \cdot (T_w - T_\infty)$$

$$T_w = \text{✓} \quad \dot{q}_v = \text{✓}$$

[2] ([1]) does.

$$\dot{q}_v \cdot V = I^2 R$$

$$I = \text{✓ A}$$

Q

Prob 12

Electrical heater

$$\dot{Q}_{gen} = 2 \text{ kW} = 2000 \text{ W}$$

$$K_{wire} = 20 \text{ W/m} \cdot ^\circ\text{C}$$

$$D = 5 \text{ mm}$$

$$L = 70 \text{ cm}$$

$$T_w = 110^\circ\text{C}$$

Solution

$$T_{max} = T_w + \frac{q_v \cdot R^2}{4K} \quad \left(0.0025 \text{ m}\right)^2 \quad \leftarrow 0.0025$$

$\downarrow$ $\leftarrow 20$
 110

$$\dot{Q}_{gen} = q_v \cdot \pi \cdot (0.0025)^2 \cdot L \rightarrow 0.7$$

$$\therefore T_{max} = 110^\circ\text{C}$$

Ans

Radial conduction in a hollow cylinder with internal heat generation

أسطوانة مجوفة مع توليد الحرارة الداخلي

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_v}{K} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = -\frac{q_v}{K}$$

$$r \frac{d^2 T}{dr^2} + \frac{dT}{dr} = -\frac{q_v}{K} r$$

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = -\frac{q_v}{K}$$

$$r \cdot \frac{dT}{dr} = -\frac{q_v}{K} \frac{r^2}{2} + C_1$$

$$\frac{dT}{dr} = -\frac{q_v \cdot r}{2K} + \frac{C_1}{r} \rightarrow [1]$$

$$T = \frac{-q_v \cdot r^2}{4K} + C_1 \ln r + C_2 \rightarrow [2]$$

B. conditions at $r=R_1 \Rightarrow T=T_{max}$, $\frac{dT}{dr} = 0$

at $r=R_2 \Rightarrow T=T_{w_0}$

المعوض في [1]

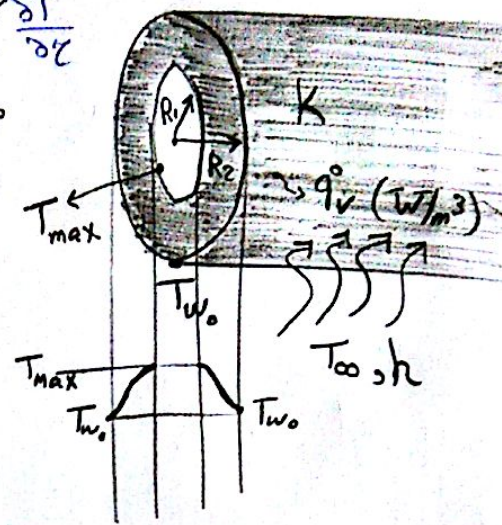
$$\frac{-q_v \cdot R_1}{2K} + \frac{C_1}{R_1} = 0$$

$$C_1 = \frac{q_v \cdot R_1^2}{2K}$$

المعوض في [2]

$$T_{w_0} = \frac{-q_v \cdot R_2^2}{4K} + \frac{q_v \cdot R_1^2}{2K} \ln R_2 + C_2$$

$$C_2 = T_{w_0} + \frac{q_v \cdot R_2^2}{4K} - \frac{q_v \cdot R_1^2}{2K} \ln R_2$$



القطر bore

F بالترتيب من c_2 الى c_1

$$T = \frac{-q_v \cdot r^2}{4K} + \frac{q_v \cdot R_1^2}{2K} \ln r + T_{w_0} + \frac{q_v \cdot R_2^2}{4K} - \frac{q_v \cdot R_2^2}{2K} \ln R_2$$

$$T = T_{w_0} + \frac{q_v \cdot R_2^2}{4K} \left[1 - \left(\frac{r}{R_2} \right)^2 \right] + \frac{q_v \cdot R_1^2}{2K} \ln \left(\frac{r}{R_2} \right)$$

Temperature distribution

at $\underline{r=R_1}$

$$T_{\max} = T_{w_0} + \frac{q_v \cdot R_2^2}{4K} \left[1 - \left(\frac{R_1}{R_2} \right)^2 \right] + \frac{q_v \cdot R_1^2}{2K} \ln \left(\frac{R_1}{R_2} \right)$$



Prob 13] A nuclear fuel element in the form of hollow cylinder - Insulated at inner surface.

No 5] نقس

$$R_1 = 5 \text{ cm} \quad \& \quad R_2 = 10 \text{ cm} \quad \& \quad T_{\infty} = 50^\circ \text{C}$$

$$k = 100 \text{ W/m}^\circ\text{C} \quad \& \quad h_o = 33 \text{ W/m}^2\text{C}$$

→ The unit surface conductance

Find $\dot{q}_v = ? \text{ W/m}^3 \quad \& \quad T_{\max} = 200^\circ \text{C}$

solution

$$T_{\max} = T_{w_o} + \frac{\dot{q}_v \cdot R_2^2}{4k} \left[1 - \left(\frac{R_1}{R_2} \right)^2 \right] + \frac{\dot{q}_v \cdot R_1^2}{2k} \cdot \ln \left(\frac{R_1}{R_2} \right) \rightarrow [1]$$

For steady flow of heat.

$$\dot{Q}_{\text{gen}} = \dot{Q}_{\text{conv}}$$

$$\dot{q}_v \cdot \pi (R_2^2 - R_1^2) \cdot L = h \cdot 2\pi R_2 \cdot L \cdot (T_{w_o} - T_{\infty}) \rightarrow [2]$$

$\dot{q}_v$ (21) π R_2^2 (0.1)² $-$ R_1^2 (0.2)² $\cdot$ L $=$ h (33) $\cdot$ 2π R_2 (0.2) $\cdot$ L $\cdot$ $(T_{w_o} - T_{\infty})$ (21) (50)

$$\dot{q}_v = \text{W/m}^3 \quad T_{w_o} = \text{W}$$

بحل [1] مع [2]

h

Prob [14]

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{q_v}{K} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = -\frac{q_v}{K}$$

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = -\frac{q_v \cdot r^2}{K}$$

$$r^2 \frac{dT}{dr} = -\frac{q_v}{3K} r^3 + c_1$$

$$\boxed{\frac{dT}{dr} = -\frac{q_v \cdot r}{3K} + \frac{c_1}{r^2}} \rightarrow [1]$$

$$\boxed{T = -\frac{q_v \cdot r^2}{6K} - \frac{c_1}{r} + c_2} \rightarrow [2]$$

التعويض في [1]

$$r=0 \rightarrow \frac{dT}{dr} = 0 \rightarrow c_1 = 0$$

التعويض في [2]

$$r=0 \rightarrow T = T_{max} \rightarrow c_2 = T_{max}$$

$$r=R \rightarrow T = T_w$$

$$T_w = -\frac{q_v \cdot R^2}{6K} + c_2 \rightarrow c_2 = T_w + \frac{q_v \cdot R^2}{6K}$$

$$T = -\frac{q_v \cdot r^2}{6K} + T_w + \frac{q_v \cdot R^2}{6K} = T_w + \frac{q_v \cdot R^2}{6K} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

$$\boxed{T_{max} = T_w + \frac{q_v \cdot R^2}{6K}}$$

